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CALCULATION OF NEAR-FREE-MOLECULAR FLUX DISTRIBUTION TO SIMPLE BODIES IN HYPERVELOCITY FLOW

John T. Miller

ARO, Inc.

March 1967

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NOTICES

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FOREWORD

The research reported herein was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), under Program Element ~~65402234~~. 62405334/8953

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This technical report has been reviewed and is approved.

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ABSTRACT

The calculations presented herein are based on a first-collision model which allows collisions between the free-stream molecules and the molecules which are re-emitted from the surface of the body. The net effect of these collisions is to partially shield the body from the free stream, reducing both the drag and heat-transfer coefficients from the corresponding values experienced in free-molecule flow. Free-stream Mach number is taken to be essentially infinite and the molecules are assumed to be re-emitted from the body surface at the most probable velocity, instead of possessing a velocity distribution. These assumptions enable the distribution of flux to a given body to be expressed as a function of the degree of rarefaction, as represented by the appropriate Knudsen number. This Knudsen number is composed of a characteristic body dimension and the mean free path of the re-emitted molecules relative to free-stream molecules. The integral equation expressing the incident flux distribution on a general body is developed, and solutions are presented for the disk normal to the free stream and for sharp cones of various apex angles, at zero angle of attack. These flux distributions are then integrated to give drag coefficients ratioed to the corresponding free-molecule values.

CONTENTS

APPENDIXES

NOMENCLATURE

A	Area
$m^{a}n$	Total shielding of point located at m resulting from a unit flux density re-emitted from a concentric ring of radius r
b	Nondimensional distance from point S_1 to point S in plane of disk
C	Constant in Eq. (1)

C_D	Drag coefficient
D	Diameter
DA	Total shielding of point located at m resulting from incremental element of area located at S
d	Distance along cone surface from apex to S
d_1	Distance along cone surface from apex to S_1
E_n	Exponential integral of order n
G	Factor defined in Eq. (21)
Kn	Knudsen number
ℓ	Length defined in Section 2.1
M	Total number of incremental areas used for calculation
M_∞	Free-stream Mach number
m	Denotes particular point for which the flux density is desired
n	Denotes particular point from which molecule is re-emitted
P	Point defined in Section 2.1
p	Pressure
$\dot{Q}$	Total heating rate
$\dot{q}$	Heat-transfer rate per unit area
R	Characteristic body dimension
r	Radial location of point S
r_1	Radial location of point S_1
Re_∞	Free-stream Reynolds number
S	Location on body from which molecule is re-emitted
S_1	Location on body which is shielded by molecule re-emitted from S
T	Temperature
U_∞	Free-stream velocity
x	Distance defined in Section 2.1
XI	Shielding of point at origin of a circular segment due to the presence of the circular segment of area

XL	Minimum value of x from which S can be "seen"
y	Ratio of density striking body to free-stream density
Z	R/λ^2
z	Length defined in Section 2.3
β	Angle defined in Section 2.2
γ	Ratio of specific heats
η	Molecular number density
η_i	Number density surviving to strike body surface
θ_c	Cone half-angle
λ	Mean free path
ρ	Density
σ	Angle defined in Section 2.1
τ	Shear
ϕ	Angle defined in Section 2.1
ψ	Collision number density per unit time
ω	Angle defined in Section 2.3

SUBSCRIPTS

FM	Free molecule
I	Inviscid
W	Wall
∞	Free stream

SECTION I
INTRODUCTION

This paper is addressed to the calculation of the interaction of a gas flow with a body in the transition flow regime between continuum and free-molecule flow. Several attempts have been made to extend continuum-flow calculations into the transition regime, the best known of which are perhaps by Van Dyke (Ref. 1) and Cheng (Ref. 2). Cheng has solved exactly a set of simplified equations which approximately describe the flow field. The model used becomes less appropriate as higher Knudsen numbers are approached, but the results obtained approach the correct limit for free-molecular flow. Van Dyke's second-order theory is valid for some portion of the transitional regime, close to continuum flow, but becomes progressively more inaccurate as the flow becomes more rarefied. In principle a "third" or higher order theory could be proposed to extend the region of validity to higher Knudsen numbers, but the formulation of these higher order terms becomes very unwieldly and then there is no assurance that they would tend to converge.

All of the aerodynamic forces acting on a body, as well as the gross heating effects, are given by the summation of the local interactions of the flow with the body surface. . These local interactions can be examined from the microscopic point of view as local collision processes between the wall and the molecules of gas in the vicinity of the wall. If the velocity distribution and the interaction potential between the molecules are known, the local flux of momentum and energy to the wall can be determined.

The problem with this approach is that the state of the gas molecules in the vicinity of the wall is not known and cannot easily be determined except in the special case of free-molecule flow. In this report the solution to the problem is formulated in such a manner that the transition regime is entered from the free-molecule flow regime.

The solution presented herein is based on a first-collision model, developed in Ref. 3, which allows collisions between the free-stream molecules and the molecules which are re-emitted from the surface of the body. After a collision, the participating molecules are considered lost, in that subsequent events are disregarded, so the net effect is a partial shielding of the body from the free-stream molecules by the molecules re-emitted from the surface.

The effectiveness of this shielding is a function of the degree of rarefaction of the flow which is expressed in terms of an appropriate Knudsen

number. This Knudsen number is based on the mean free path of the re-emitted molecules relative to collisions with the free-stream molecules. This single mean free path value is determined by assuming that all incident molecules are re-emitted from the surface at the most probable velocity instead of possessing a velocity distribution.

Once the distribution of incident free-stream molecules on the surface of a body is determined, the pressure distribution and heat-transfer distribution can be found. These distributions can then be integrated to yield drag coefficient and total heat flux to the body in near-free-molecule flow.

For any experimental data a problem arises concerning the determination of the effective Knudsen number, Kn, which evolves naturally in the theory. Reference 3 gives the result

$$Kn = C\gamma^{1/2} M_\infty / Re_\infty \tag{1}$$

where Kn is the Knudsen number, M_∞ is the free-stream Mach number, Re_∞ is the free-stream Reynolds number, and C is a "free constant" which is chosen to correlate the experimental data with theory. This approach yields a curve which passes through the data, whose shape has theoretical justification, and whose value has an experimental basis. While this approach is not completely satisfactory from a theoretical viewpoint, its justification can be argued on the basis that this constant is necessary to account for the uncertain force field which exists between the actual molecules. For example, for billiard ball molecules this constant has a theoretical value of C = 1.26 when calculating free-stream Knudsen number, Kn_∞. In Ref. 3, which deals with hypersonic spheres in near-free-molecular flow, it was found that C = 2.446.

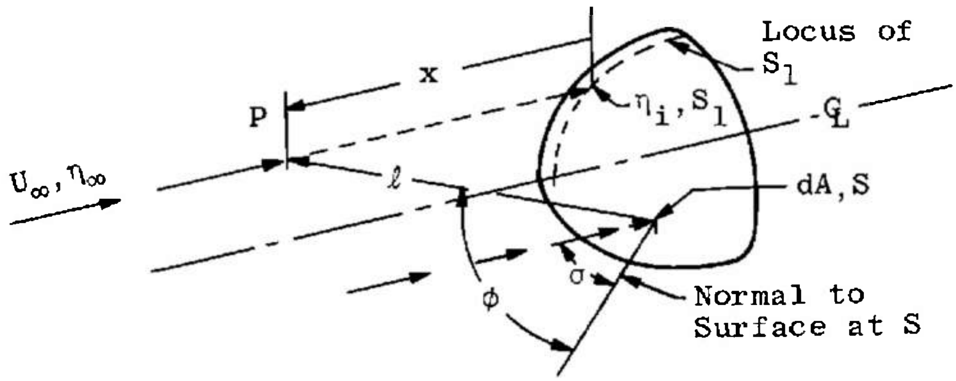
The results presented herein are given in terms of the rarefaction parameter, R/λ, where R is a characteristic body dimension and λ is the mean free path of the re-emitted molecules in the uniform free stream. These results are strictly applicable only for small values of R/λ. However, the theory gives the proper limit for flow with large R/λ in the case of a blunt body.

SECTION II
ANALYSIS

2.1 GENERAL AXISYMMETRIC BODIES

The following development assumes that the free stream is composed of molecules having a common direction and velocity, which is equivalent

to assuming an infinite Mach number. Consider the general body immersed in such a stream as sketched below.



For a streamwise tube of unit cross-sectional area terminating at the body surface centered on point S_1, the difference in number density of free-stream molecules entering the tube and the number density of molecules actually surviving to strike the surface of the model is given by

$$\left[\eta_\infty - \eta_i(S_1)\right] U_\infty = \int_0^\infty \psi \, dx \tag{2}$$

where η_∞ and η_i (S_1) are the number density of molecules at infinity and the number density striking the surface of the model, respectively, and ψ is the collision number density per unit time at P. To find ψ, consider the contribution of the events at dA on the surface to the events at the point P. Then one sees that

$$d\psi = U_\infty \eta_i(S) \cdot \cos\sigma \, dA \cdot \frac{\cos\phi}{\pi} \cdot \frac{1}{\ell^2} \cdot e^{-\ell/\lambda} \cdot \frac{1}{\lambda} \tag{3}$$

The significance of the various quantities is as follows:

$U_\infty \eta_i(S)$ = number flux impinging on dA ($\eta_i(S)$ is the number density striking the surface at location S.)

$\cos\sigma \, dA$ = unit frontal area

$\frac{\cos\phi}{\pi}$ = probability that a molecule is re-emitted along ℓ (diffuse reflection)

$\frac{1}{\ell^2}$ = decrease in density at P caused by spherical spreading of re-emitted molecules

$e^{-\ell/\lambda}$ = probability of surviving passage to P without suffering a collision

$\frac{1}{\lambda}$ = probability of collision per unit length at P

$d\psi$ = collisions per unit volume per unit time

ψ is then given by integration over the portion of the body surface which can be "seen" from P. Thus,

$$\left[\eta_\infty - \eta_i(S_1)\right] U_\infty = \frac{1}{\pi\lambda} \int^{\text{area of body}} \int_{\cos\phi=0}^{x=\infty} \cos\sigma \; U_\infty \eta_i(S) \frac{\cos\phi}{\ell^2} e^{-\ell/\lambda} \, dx \, dA \tag{4}$$

Defining $y = \eta_i/\eta_\infty$, then

$$y(S_1) = 1 - \frac{1}{\pi\lambda} \int^{\text{area of body}} y(S) \cos\sigma \int_{\cos\phi=0}^{x=\infty} \frac{\cos\phi}{\ell^2} e^{-\ell/\lambda} \, dx \, dA \tag{5}$$

where S denotes the location on the body surface from whence the molecules are emitted and S_1 denotes the location on the body from which the x vector is drawn, that is, the location on the body surface which is partly shielded from the free stream by collisions between molecules from S and the free stream.

Equation (5) is valid for any body configuration, and the solution gives the flux density which impinges on the point S_1 on the body surface. The inner integral is a function of particular body geometry and the location of S_1 and S. This equation is a nonhomogenous Fredholm linear integral equation of the second kind (Ref. 4). The solution was obtained by dividing the forward-facing surface of the body into small increments of area such that y(S) over each small area could be taken to be constant. This allows y(S) to be moved outside the integral and Eq. (5) can be rewritten as

$$y(m/M) = 1 - \frac{1}{\pi\lambda} \sum_{n=0}^{M} y(n/M) \int^{\Delta A(n)} \cos\sigma \int_{\cos\phi=0}^{x=\infty} \frac{\cos\phi}{\ell^2} e^{-\ell/\lambda} \, dx \, dA \tag{6}$$

where M is the total number of area increments used, and n and m denote the particular area increments. If ${}_m a_n$ is defined by

$${}_m a_n = \frac{1}{\pi\lambda} \int^{\Delta A(n)} \cos\sigma \int_{\cos\phi=0}^{x=\infty} \frac{\cos\phi}{\ell^2} e^{-\ell/\lambda} \, dx \, dA$$

then the equation can be expressed as the following array:

$$y(0/M) = 1 - {}_0a_0\, y(0/M) - {}_0a_1\, y(1/M) - {}_0a_2\, y(2/M) - {}_0a_3\, y(3/M) \cdots {}_0a_M\, y(1)$$

$$y(1/M) = 1 - {}_1a_0\, y(0/M) - {}_1a_1\, y(1/M) - {}_1a_2\, y(2/M) - {}_1a_3\, y(3/M) \cdots {}_1a_M\, y(1)$$

$$\vdots$$

$$y(1) = 1 - {}_Ma_0\, y(0/M) - {}_Ma_1\, y(1/M) - {}_Ma_2\, y(2/M) - {}_Ma_3\, y(3/M) \cdots {}_Ma_M\, y(1)$$

or,

$$\left(1 - {}_0a_0\right) y(0/M) + {}_0a_1\, y(1/M) + {}_0a_2\, y(2/M) - {}_0a_3\, y(3/M) + \cdots {}_0a_M\, y(1) = 1$$

$${}_1a_0\, y(0/M) + \left(1 + {}_1a_1\right) y(1/M) + {}_1a_2\, y(2/M) + \cdots\cdots {}_1a_M\, y(1) = 1$$

$$\vdots$$

$${}_Ma_0\, y(0/M) + {}_Ma_1\, y(1/M) + \cdots\cdots \left(1 + {}_Ma_M\right) y(1) = 1$$

Thus the solution is given by

$$\begin{pmatrix} y(0) \\ y(1/M) \\ y(2/M) \\ \vdots \\ y(1) \end{pmatrix} = \begin{pmatrix} \left(1 + {}_0a_0\right) & {}_0a_1 & {}_0a_2 & \cdots & {}_0a_M \\ {}_1a_0 & \left(1 + {}_1a_1\right) & {}_1a_2 & \cdots & {}_1a_M \\ {}_2a_0 & {}_2a_1 & \left(1 + {}_2a_2\right) & \cdots & {}_2a_M \\ & & & & \\ {}_Ma_0 & {}_Ma_1 & {}_Ma_2 & \cdots & \left(1 - {}_Ma_M\right) \end{pmatrix}^{-1} \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \quad (7)$$

In principle the solution could be obtained in this manner for any body geometry, but practically the number of points is limited by the size of the M by M ${}_ma_n$ matrix to be inverted.

The a's actually represent the influence of the n location on the m location on the body. For both free-molecule and continuum flow, the off-diagonal elements are zero. This is because in free-molecular flow only an insignificant number of re-emitted molecules suffer a collision in a finite distance from the point, while at the continuum limit, $R/\lambda \to \infty$, all

the significant interactions occur in the immediate vicinity of the point. Thus, ${}_n a_n$ goes uniformly from zero at $R/\lambda \to 0$ to unity at $R/\lambda \to \infty$, giving, for any body geometry, $y \to 1.0$ for free-molecule flow and $y \to 0.5$ for $R/\lambda \to \infty$.

It is obvious that the largest coefficient will come from the consideration of the effect of an element upon itself. That is, when m and n coincide. In this situation, $\ell \to x$ and $\cos\phi \to \cos\sigma$, and taking $\cos\sigma\, dA$ as the incremental normal area, dA_n, the contribution to ${}_n a_m$ becomes

$$\frac{1}{\pi\lambda}\int^{\Delta An}\int_0^{x\to\infty} \cos\sigma\, \frac{e^{-x/\lambda}}{x^2}\, dx\, dA_n$$

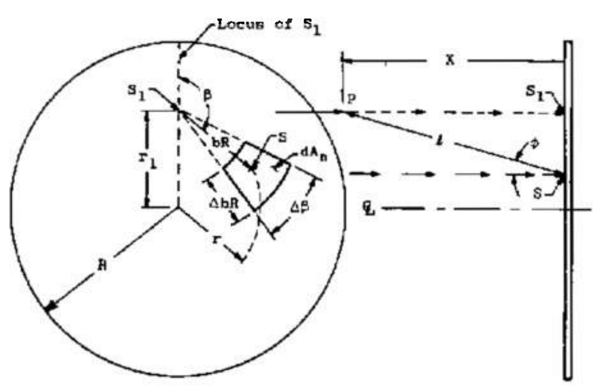
in which the integral is unbounded for $x \to 0$. The remedy is to choose a coordinate system for dA_n such that $dA_n \to 0$ at the point in question. A cylindrical coordinate system with the origin at the point in question satisfies this requirement and gives a finite result for ${}_n a_m$. For a more complete discussion, the problem must be expressed in a particular coordinate system.

2.2 FLAT DISK NORMAL TO FLOW

The coordinate system used for this case is cylindrical, with the axis shifted to the point S_1 at which the flux density is desired so that the incremental normal area, dA_n, approaches zero as S and S_1 coalesce. Using the local nondimensional radius, r_1/R for S_1 and r/R for S, the equation to be evaluated is

$$y(r_1/R) = 1 - \frac{1}{\pi\lambda}\int^{\text{Normal Area}} y(r/R) \int_{\cos\phi=0}^{\infty} \frac{\cos\phi}{\ell^2}\, e^{-\ell/\lambda}\, dx\, dA_n \tag{8}$$

with the symbols defined by the sketch below.



For this body shape the following relations may be found:

$$\cos\phi \; dx = d\ell$$

$$dA_n = R^2 \, b \, db \, d\beta$$

and as $\cos\phi \longrightarrow 0$, $x \longrightarrow 0$ and $\ell = Rb$. Using these relations and assigning the appropriate limits of integration, the problem becomes the solution of

$$y(r_1/R) = 1 - \frac{2R^2}{\pi\lambda} \int_{\beta=0}^{\pi} \int_{b=0}^{b=\frac{r_1}{R}\cos\beta + \sqrt{1-(r_1/R)^2 \sin^2\beta}} y(r/R)\, b \times \int_{\ell=Rb}^{\infty} \frac{e^{-\ell/\lambda}}{\ell^2} \, d\ell \, db \, d\beta \tag{9}$$

where

$$(r/R) = \sqrt{(r_1/R)^2 + b^2 - 2(r_1/R)\, b \cos\beta}$$

Now considering the innermost integral, one finds

$$\int_{\ell=Rb}^{\infty} \frac{e^{-\ell/\lambda}}{\ell^2} \, d\ell = \frac{1}{Rb} \int_{1}^{\infty} \frac{e^{-\frac{R}{\lambda} bz}}{z^2} \, dz = \frac{1}{Rb} E_2 \left(\frac{R}{\lambda} b \right)$$

where E_n denotes the exponential integral of order n. Using this result, Eq. (9) becomes

$$y(r_1/R) = 1 - \frac{2}{\pi} \frac{R}{\lambda} \int_{\beta=0}^{\pi} \int_{b=0}^{b=(r_1/R)\cos\beta + \sqrt{1-(r_1/R)^2 \sin^2\beta}} y(r/R)\, E_2 \left(\frac{R}{\lambda} b \right) db \, d\beta$$

If $y(r/R)$ is approximately constant for a small interval in b,

$$b - \frac{\Delta b}{2} \leq b \leq b + \frac{\Delta b}{2}$$

then

$$\int_{b-\frac{\Delta b}{2}}^{b-\frac{\Delta b}{2}} y(r/R)\, E_2 \left(\frac{R}{\lambda} b \right) db \approx y(r/R)\Big|_b \frac{\lambda}{R} \int_{(R/\lambda)\left(b-\frac{\Delta b}{2}\right)}^{(R/\lambda)\left(b+\frac{\Delta b}{2}\right)} E_2(x)\, dx$$

$$= y(r/R)\Big|_b \frac{\lambda}{R} \left\{ E_3 \left[\frac{R}{\lambda} \left(b - \frac{\Delta b}{2} \right) \right] - E_3 \left[\frac{R}{\lambda} \left(b + \frac{\Delta b}{2} \right) \right] \right\} \tag{10}$$

The contribution of area $dA_n = R^2 \, b \, db \, d\beta$ located at S to the shielding of point S_1, is given by

$$y(r/R)_S \; DA = \frac{2}{\pi} \Delta\beta \; y(r/R)_S \left\{ E_3 \left[(R/\lambda) \left(b - \frac{\Delta b}{2} \right) \right] - E_3 \left[(R/\lambda) \left(b + \frac{\Delta b}{2} \right) \right] \right\} \quad (11)$$

which defines DA.

Let $r_1/R = S_1 = 0.1m$ and $r/R = S = 0.1n$, where n and m are integers which vary between 0 and 10. (This assumes that the disk is divided into ten concentric rings on each of which y(s) = constant.) The coefficient $_m a_n$ can then be defined as

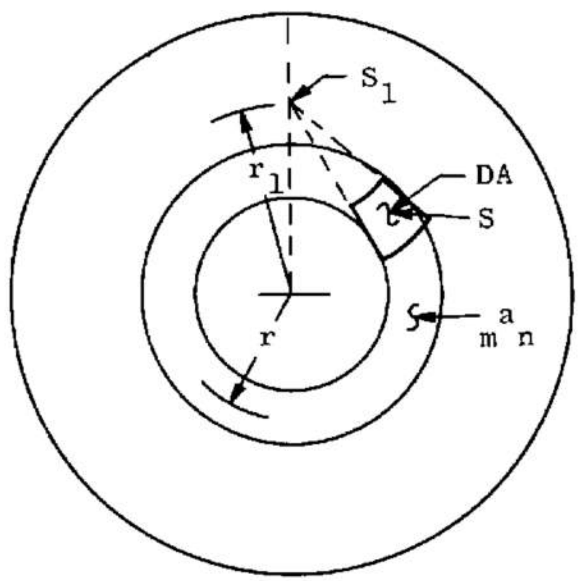
$$_m a_n = \Sigma \; DA \quad (12)$$

with

$$r = \text{constant} = 0.1n$$

$$r_1 = \text{constant} = 0.1m$$

This nomenclature is clarified by the following sketch:

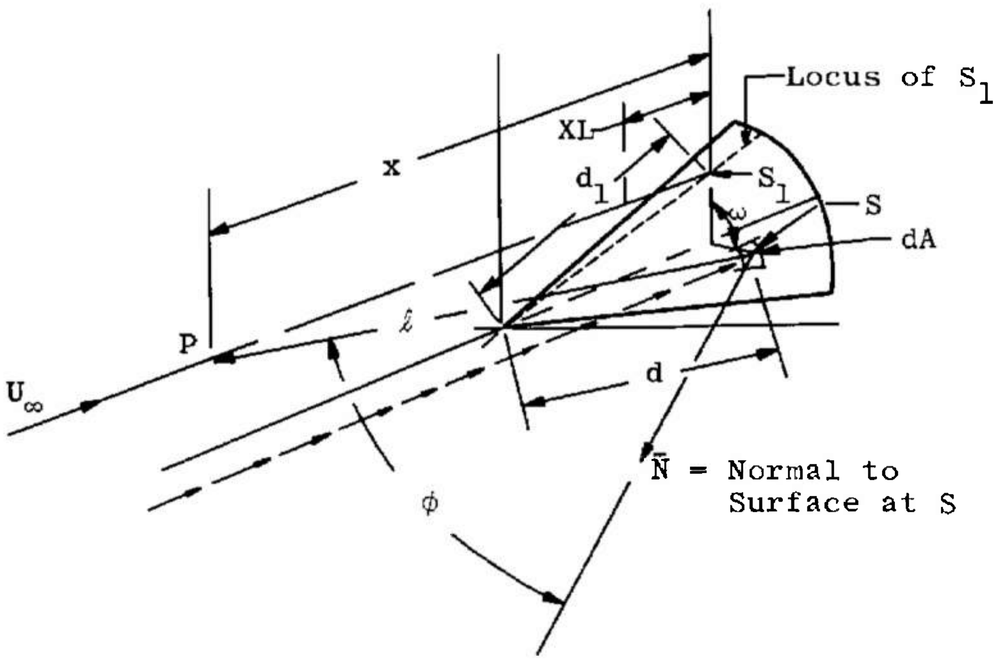


The frontal area of the disk is covered by small units of area as shown, which are defined by a cylindrical coordinate system with the origin located at S_1. The corresponding DA is calculated and its contribution is credited to the proper $_m a_n$. The location of S_1 is then changed, so that a complete set of $_m a_n$ coefficients is obtained, with m and n taking values from zero to ten. This gives an 11- by -11 matrix, the inversion of which yields the flux density impinging on the body at eleven evenly spaced radii.

A simple extension of the calculation has been included to give the influence of the presence of the disk on the fluid flow in its plane beyond the edge of the disk.

2.3 SHARP-NOSED CONES

For this configuration, a cylindrical coordinate system is used, with the coordinate system centered on the cone axis. This arrangement is suitable for all cases in which the points S and S_1 do not coalesce. When the points S and S_1 do coalesce, the origin of the coordinate system is shifted to the point itself, but only for the calculation of the influence of flow interactions in the immediate vicinity of this point. That is, after this particular DA is calculated, the coordinate system origin is immediately shifted back to the cone axis, and the calculation proceeds. The pertinent geometry is shown below.



Here, S_1 denotes the location of the point at which the shielding is desired because of the influence of the area dA located at S. The angle ϕ is the angle between the normal to the surface (at S) and ℓ, the path of particles emitted from dA which may collide at P with free-stream particles. The quantity XL is the lower limit on the x-integration (for $x \rightarrow XL$, S_1 can no longer "see" S). When $\cos\phi = 0$, $x = XL$.

Taking the cone half-angle as θ_c, the necessary quantities are as follows:

$$\ell^2 = [x - \cos\theta_c (d_1 - d)]^2 + \sin^2\theta_c (d_1^2 + d^2 - 2dd_1 \cos\omega)$$

$$XL = d_1 \cos\theta_c (1 - \cos\omega)$$

$$\cos\phi = \frac{\sin\theta_c}{\ell} [x - d_1 \cos\theta_c (1 - \cos\omega)] \tag{13}$$

where ω is the angle between the plane through the cone axis containing S_1 and a plane containing both S and the axis. These relations in Eq. (13) are used to yield

$$y(S_1) = 1 - \frac{1}{\pi\lambda} \int^{\text{Normal Area}} y(S) \int_{XL}^{\infty} \cos\phi \frac{e^{-\ell/\lambda}}{\ell^2} dx\, dA_n \tag{14}$$

where

$$dA_n = d \sin\theta_c\, d\omega\, d(d \sin\theta_c)$$

It is convenient to nondimensionalize Eq. (14) by the cone base radius, R, which is given by $R = S_{max} \sin\theta_c$. Using this result and retaining the previous symbols for the new nondimensional quantities, the equation for DA becomes (when S and S_1 do not coalesce)

$$DA_{(S \neq S_1)} = \frac{2}{\pi} \Delta\omega \frac{R}{\lambda} (d) (\Delta d) \int_{XL}^{\infty} \cos\phi \frac{e^{-(R/\lambda)\ell}}{\ell^2} dx \tag{15}$$

This integral can be evaluated numerically with reasonable accuracy using a computer, provided the integration is performed over an appropriate interval. The procedure used was to calculate approximately the appropriate interval, integrate the function through this interval and then check to see if further integration was required. Finally, the value of the integral from this upper limit to infinity was approximated, using

$$\cos\phi \approx \sin\theta_c, \; dx \approx d\ell$$

and

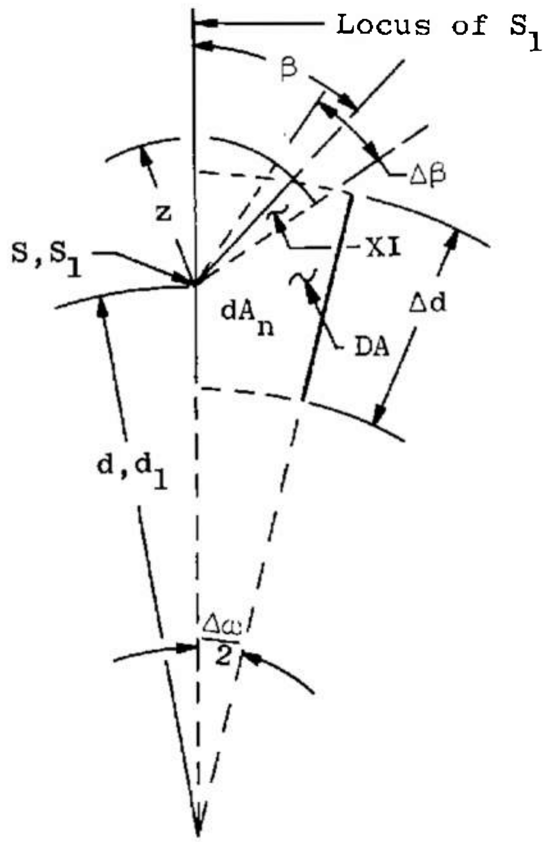
$$\int_{x_{upper}}^{\infty} \cos\phi \frac{e^{-(R/\lambda)\ell}}{\ell^2} dx \approx \int_{\ell_{upper}}^{\infty} \sin\theta_c \frac{e^{-(R/\lambda)\ell}}{\ell^2} d\ell = \frac{\sin\theta_c}{\ell_{upper}} \int_1^{\infty} \frac{e^{-(R/\lambda)z\,\ell_{upper}}}{z^2} dz$$

$$= \frac{\sin\theta_c}{\ell_{upper}} E_2 \left[(R/\lambda)\, \ell_{upper}\right] \tag{16}$$

Thus,

$$DA_{(S \neq S_1)} = \frac{2}{\pi} \Delta\omega \left(\frac{R}{\lambda}\right) d\,(\Delta d) \left[\int_{XL}^{x_{upper}} \cos\phi \, \frac{e^{-(R/\lambda)\ell}}{\ell^2} \, dx + \frac{\sin\theta_c}{\ell_{upper}} E_2 \left(\frac{R}{\lambda} \ell_{upper}\right)\right] \tag{17}$$

When S and S_1 coalesce, the shielding of the point by its own re-emitted molecules is computed by assuming the element of normal area, dA_n, is actually oriented normal to the free stream. This unit of area is subdivided into smaller units as shown in the sketch, which shows an axial view of the cone. A cylindrical coordinate system is used with the



origin at the points S and S_1. The angle β is varied between 0 and 180 deg by equal increments of $\Delta\beta$, the appropriate z is calculated, and the local contribution to DA for each value of β is calculated using the expression

$$XI = \frac{2}{\pi} \Delta\beta \left[0.5 - E_3\left(\frac{R}{\lambda} z\right)\right] \tag{18}$$

where the factor 0.5 = E_3 (0). The factor DA is then given by

$$DA_{(S = S_1)} = \frac{2}{\pi} \Delta\beta \sum_{\beta=0}^{180°} \left[0.5 - E_3(Z)\right] \tag{19}$$

These DA's are next summed, as previously done for the disk, to give the ${}_m a_n$ quantities, viz,

$$ {}_m a_n = \Sigma \, DA $$

with

$$ S = \text{constant} = 0.1n $$

$$ S_1 = \text{constant} = 0.1m $$

Once these are found, the solution for $y(S_1)$ is given by the matrix inversion technique used for the disk.

SECTION III
DISCUSSION AND CONCLUSIONS

The results of the disk calculation are shown in Fig. 1. Also shown for comparison are the flux densities for free-molecule flow ($R/\lambda \rightarrow 0$) and continuum flow ($R/\lambda \rightarrow \infty$). As would be expected, the greater shielding occurs at the axis of the disk, and decreases with distance from the axis. One interesting result is that the shielding at a given distance beyond the edge of the disk is a maximum for a particular value of R/λ which depends upon the distance from the disk.

A typical result of the cone calculation is shown in Fig. 2 for $R/\lambda = 5.0$. The validity of the flow model used for the calculation is probably questionable for this relatively high value of R/λ, but this particular result is used to illustrate the main features of the calculation. The most important result is that the point of the cone does not experience a free-molecule flux density except in the case of a vanishingly small cone angle. Also, except for the region adjacent to the base of the cone, the incident flux level is identical for any cone length in terms of x/λ. This indicates that most of the shielding of a point on the cone is attributable to molecules re-emitted from the body upstream of the point. Finally, the lesser shielding of a point by all locations aft of that point is related to the sudden increase in flux density near the aft end of the cone. Also shown for comparison is the corresponding result for the normal disk (θ_c = 90 deg).

Figure 3 shows the effect of varying R/λ for a particular cone angle. As would be expected, the incident flux density decreases uniformly as R/λ increases.

The results obtained for all the calculations are tabulated in Table I. The disk is taken as a cone of $\theta_c = 90$ deg. Table II gives the results for the influence of the disk in the plane of but beyond the edge of the disk.

It is of interest to interpret these results in terms of quantities useful in aerodynamics. Since in free-molecule flow, pressure, shear stress, and heat transfer are proportional to free-stream density, the tabulated quantity, local η_i/η_∞, may be interpreted in those terms, viz,

$$\eta_i/\eta_\infty = p/p_{FM} = \dot{q}/\dot{q}_{FM} = \tau/\tau_{FM}$$

The drag coefficient ratio is given by

$$C_D/C_{D_{FM}} = 1/A_n \int^{\text{Normal Area}} \eta_i/\eta_\infty \, dA_n \tag{20}$$

which is also equal to $\dot{Q}/\dot{Q}_{FM}$, the total heating rate ratioed to the free-molecular value. These results are shown in Fig. 4 as a function of R/λ. It is noted that as $R/\lambda \longrightarrow \infty$, $C_D/C_{D_{FM}} \longrightarrow 0.5$, which is approximately correct for a blunt body.

Comparison of these results with experimental data requires that the mean free path of the re-emitted molecules be defined. In Ref. 3 it is shown that

$$\lambda = \lambda_\infty \frac{G}{1 + M_\infty \sqrt{\dfrac{8\gamma}{9\pi} \dfrac{T_\infty}{T_w}}} \quad \text{where } G \approx 2 \text{ for } M_\infty \geq 6.0 \tag{21}$$

Also, Knudsen number, $\lambda_\infty/D = C\gamma^{\frac{1}{2}} M_\infty/Re_\infty$

where C is a constant and D = 2R. This gives

$$R/\lambda = \frac{Re_\infty}{2C\gamma^{\frac{1}{2}} M_\infty} \frac{\left(1 + M_\infty \sqrt{\dfrac{8\gamma}{9\pi} \dfrac{T_\infty}{T_w}}\right)}{\sqrt{2}} \tag{22}$$

An experimental value of C = 2.446 is given in Ref. 3 for spheres in a hypersonic, high-enthalpy nitrogen stream.

It is of interest to compare the present results with similar calculations by Kogan and Degtyarev published in Ref. 5. Their results are based on an approximate solution of the Boltzmann equation by the Monte Carlo method, and allow the existence of a molecular boundary layer as Kogan previously postulated in Ref. 6. A comparison between their calculations and the present results is shown in Fig. 5.

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APPENDIXES

I. ILLUSTRATIONS
II. TABLES

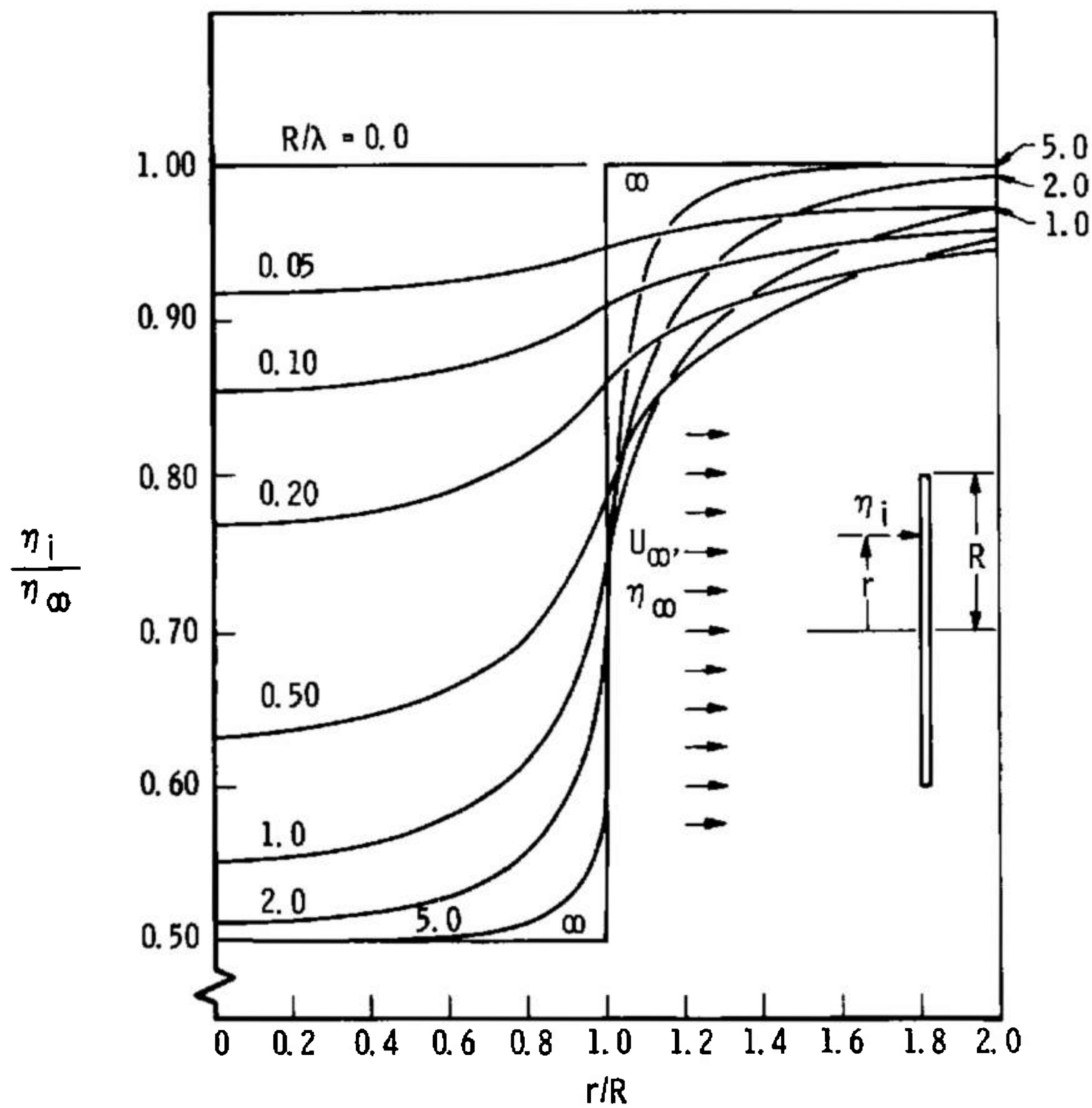

Fig. 1 Flux Distribution to Normal Disk

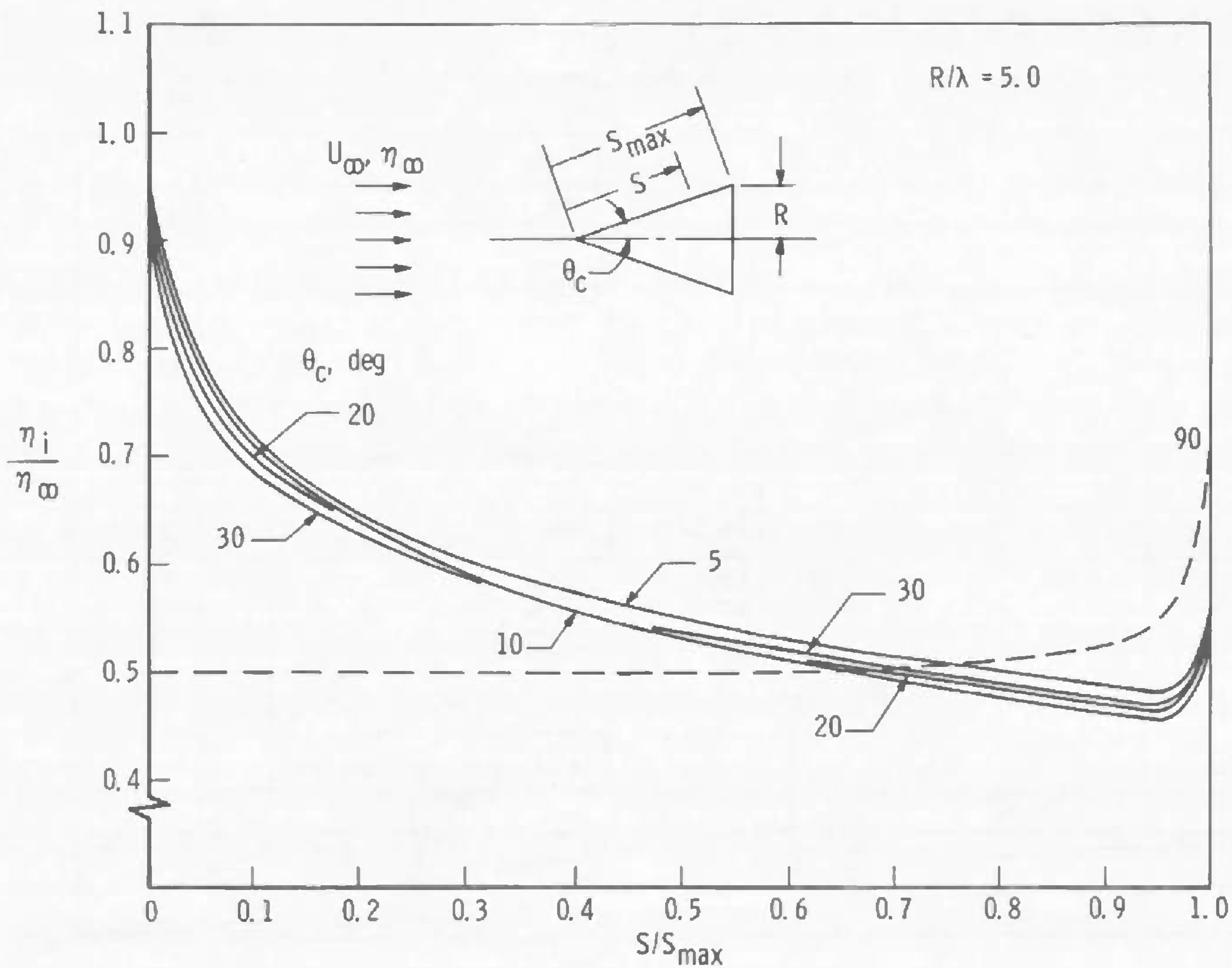

Fig. 2 Flux Distribution to Sharp Cone

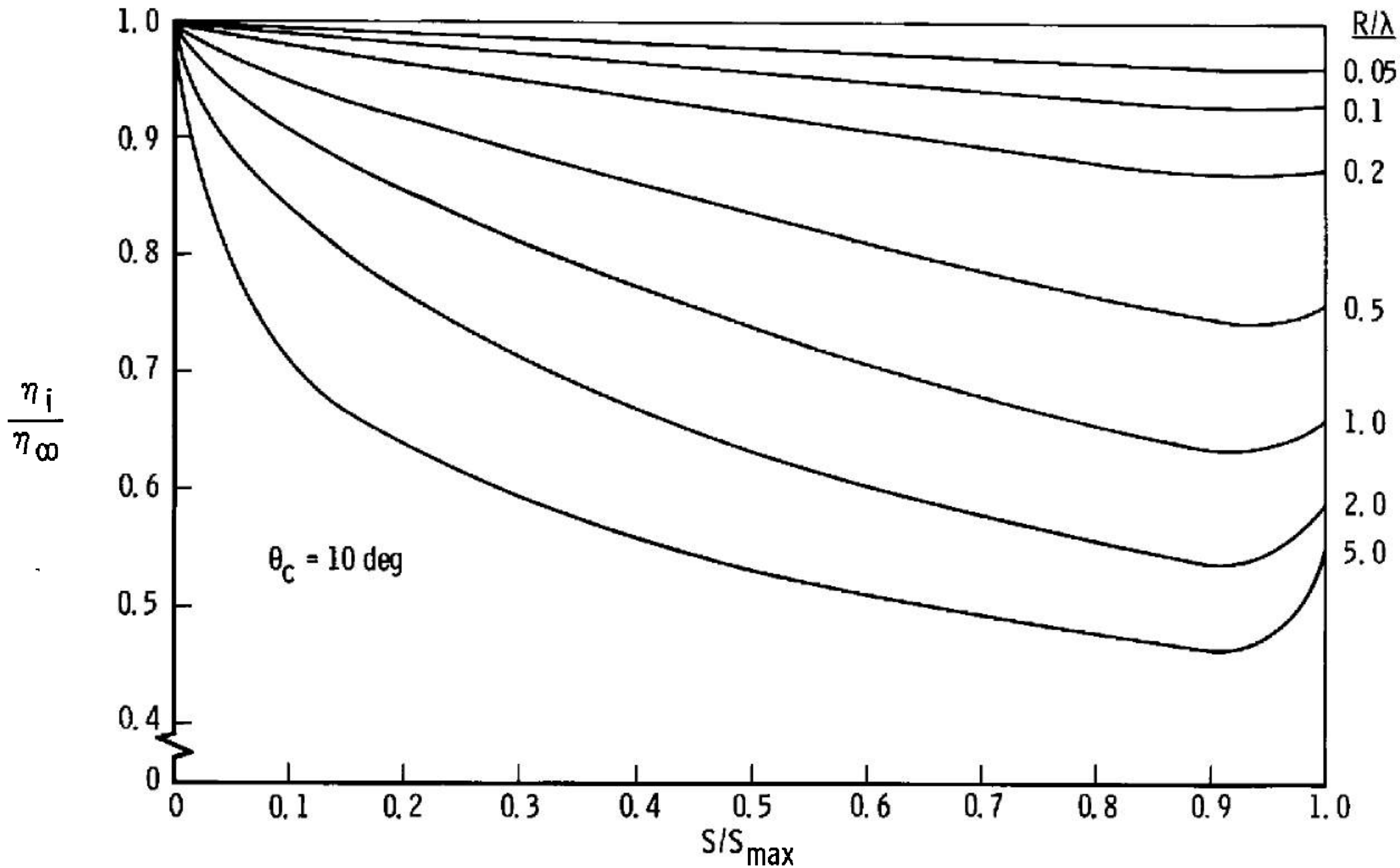

Fig. 3 Incident Flux Distribution on a 10-deg Cone

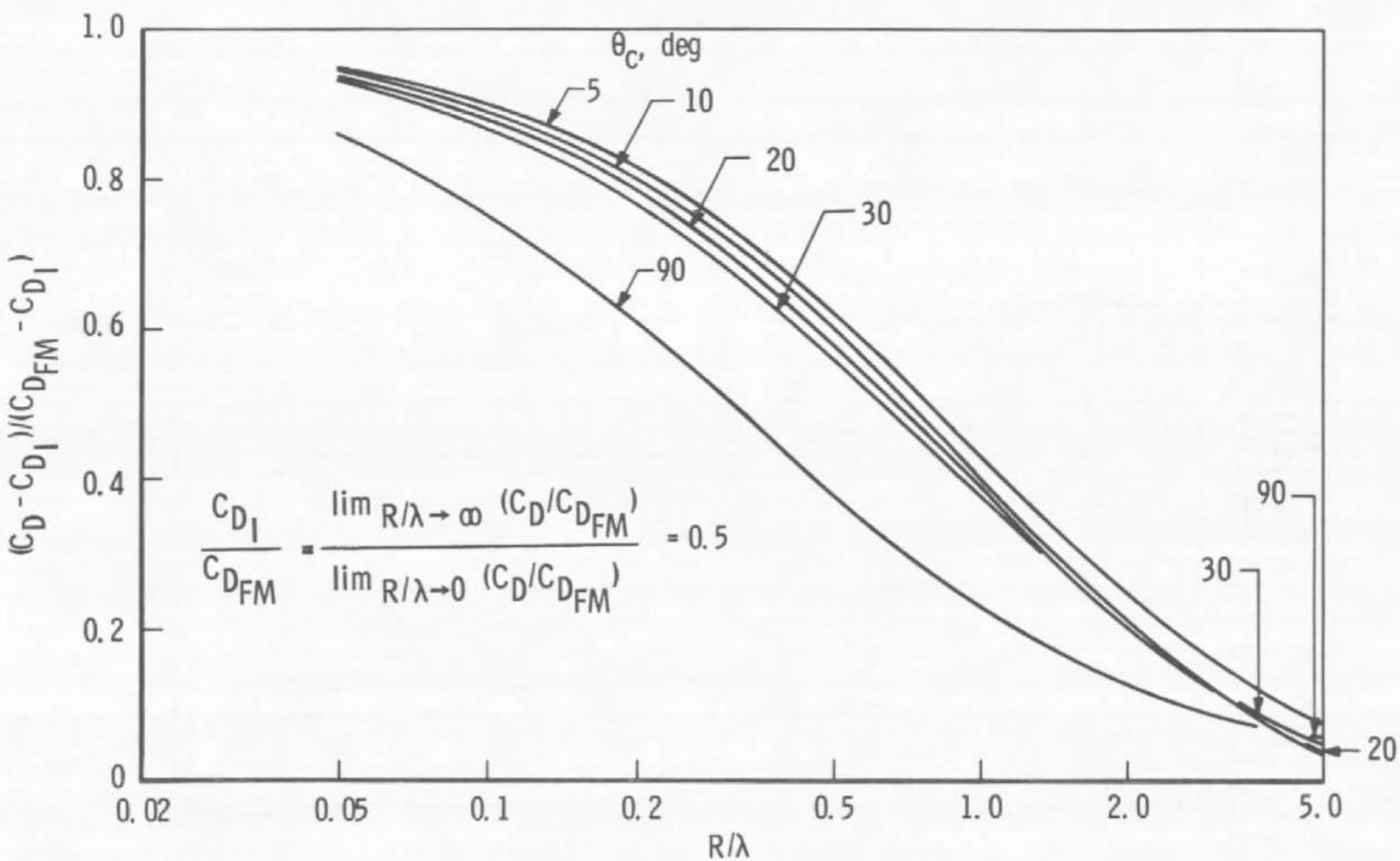

Fig. 4 Drag Coefficient for Sharp Cones

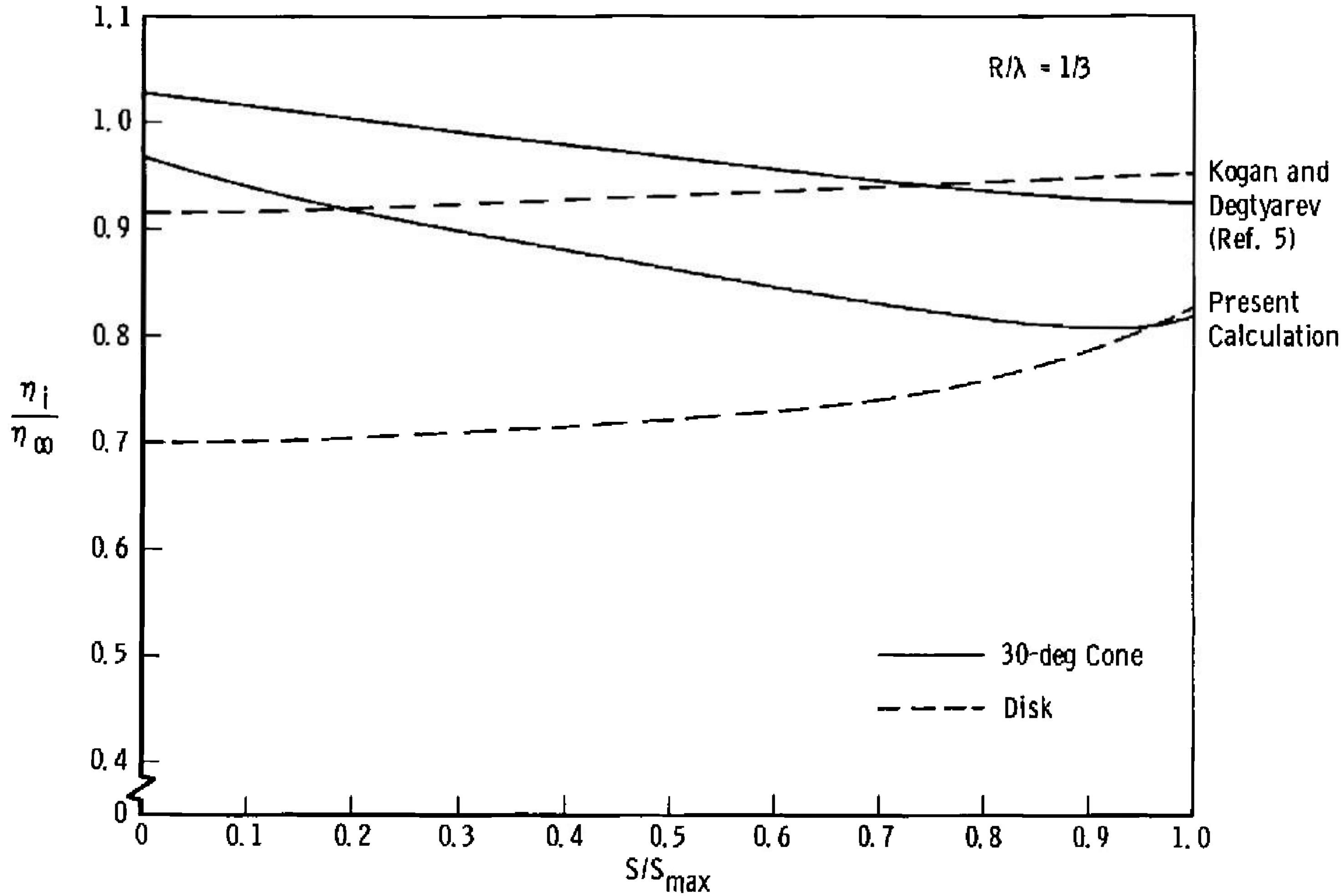

Fig. 5 Incident Flux Distributions by Two Methods

TABLE I
TABULATED VALUES OF INCIDENT FLUX DENSITY

	S/S_{max}										
θ_c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
						$R/\lambda = 0.05$					
5°	0.9996	0.9947	0.9910	0.9876	0.9837	0.9798	0.9760	0.9721	0.9820	0.9643	0.9649
10°	0.9988	0.9938	0.9899	0.9862	0.9820	0.9778	0.9737	0.9695	0.9654	0.9613	0.9618
20°	0.9953	0.9902	0.9862	0.9822	0.9780	0.9738	0.9698	0.9658	0.9619	0.9583	0.9594
30°	0.9891	0.9840	0.9801	0.9760	0.9720	0.9680	0.9644	0.9608	0.9575	0.9547	0.9567
90°	0.9166	0.9168	0.9175	0.9186	0.9201	0.9222	0.9249	0.9284	0.9328	0.9387	0.9476
						$R/\lambda = 0.1$					
5°	0.9993	0.9897	0.9824	0.9758	0.9683	0.9610	0.9538	0.9465	0.9393	0.9323	0.9338
10°	0.9982	0.9882	0.9806	0.9734	0.9655	0.9576	0.9499	0.9421	0.9345	0.9271	0.9282
20°	0.9924	0.9822	0.9746	0.9669	0.9590	0.9511	0.9437	0.9363	0.9291	0.9225	0.9248
30°	0.9818	0.9718	0.9643	0.9566	0.9490	0.9416	0.9349	0.9282	0.9222	0.9171	0.9210
90°	0.8546	0.8550	0.8561	0.8579	0.9607	9.8643	0.8691	0.8751	0.8830	0.8937	0.9102
						$R/\lambda = 0.2$					
5°	0.9988	0.9799	0.9660	0.9537	0.9399	0.9267	0.9138	0.9011	0.8887	0.8768	0.8806
10°	0.9973	0.9778	0.9633	0.9500	0.9354	0.9212	0.9075	0.8941	0.8809	0.8684	0.8716
20°	0.9888	0.9690	0.9546	0.9407	0.9262	0.9122	0.8991	0.8862	0.8740	0.8629	0.8679
30°	0.9721	0.9529	0.9390	0.9251	0.9115	0.8983	0.8867	0.8752	0.8648	0.8564	0.8641
90°	0.7668	0.7673	0.7690	0.7719	0.7761	0.7819	0.7894	0.7992	0.8121	0.8301	0.8594
						$R/\lambda = 0.5$					
5°	0.9974	0.9525	0.9218	0.8962	0.9686	0.8436	0.8202	0.7983	0.7778	0.7590	0.7718
10°	0.9956	0.9494	0.9177	0.8902	0.8611	0.8345	0.8098	0.7866	0.7649	0.7450	0.7565
20°	0.9839	0.9374	0.9067	0.8785	0.8503	0.8244	0.8011	0.7791	0.7588	0.7412	0.7555
30°	0.9586	0.9144	0.8855	0.8582	0.8321	0.8080	0.7874	0.7678	0.7506	0.7372	0.7559
90°	0.6330	0.6337	0.6360	0.6398	0.6455	0.6535	0.6642	0.6787	0.6990	0.7292	0.7866
						$R/\lambda = 1.0$					
5°	0.9952	0.9119	0.8613	0.8227	0.7830	0.7496	0.7203	0.6943	0.6710	0.6507	0.6789
10°	0.9933	0.9077	0.8559	0.8144	0.7728	0.7373	0.7065	0.6789	0.6543	0.6328	0.6587
20°	0.9800	0.8946	0.8453	0.8033	0.7638	0.7298	0.7009	0.6749	0.6518	0.6325	0.6615
30°	0.9498	0.8699	0.8244	0.7845	0.7484	0.7171	0.6915	0.6683	0.6484	0.6335	0.6675
90°	0.5526	0.5532	0.5551	0.5585	0.5636	0.5710	0.5814	0.5964	0.6191	0.6561	0.7451
						$R/\lambda = 2.0$					
5°	0.9914	0.8456	0.7745	0.7273	0.6816	0.6473	0.6194	0.5957	0.5754	0.5583	0.6099
10°	0.9893	0.8398	0.7675	0.7166	0.6690	0.6323	0.6027	0.5774	0.5558	0.5371	0.5850
20°	0.9746	0.8268	0.7595	0.7086	0.6641	0.6295	0.6017	0.5777	0.5570	0.5400	0.5906
30°	0.9409	0.8045	0.7436	0.6955	0.6556	0.6239	0.5992	0.5775	0.5591	0.5456	0.6011
90°	0.5114	0.5117	0.5126	0.5144	0.5172	0.5217	0.5285	0.5393	0.5580	0.5931	0.7277
						$R/\lambda = 5.0$					
5°	0.9826	0.7189	0.6464	0.6090	0.5729	0.5495	0.5310	0.5147	0.5001	0.4871	0.5754
10°	0.9809	0.7108	0.6379	0.5966	0.5584	0.5325	0.5121	0.4940	0.4776	0.4631	0.5447
20°	0.9651	0.7008	0.6353	0.5933	0.5584	0.5341	0.5147	0.4974	0.4816	0.4677	0.5512
30°	0.9269	0.6868	0.6287	0.5890	0.5584	0.5364	0.5188	0.5027	0.4880	0.4758	0.5634
90°	0.5002	0.5003	0.5003	0.5005	0.5009	0.5015	0.5029	0.5057	0.5130	0.5246	0.7125

TABLE II
INCIDENT FLUX DENSITY IN PLANE OF DISK

r/R =	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
R/λ = 0.05	0.9544	0.9582	0.9611	0.9636	0.9656	0.9674	0.9689	0.9702	0.9715	0.9725
R/λ = 0.1	0.9235	0.9307	0.9364	0.9410	0.9449	0.9483	0.9512	0.9538	0.9561	0.9581
R/λ = 0.2	0.8845	0.8977	0.9080	0.9163	0.9233	0.9292	0.9343	0.9388	0.9438	0.9463
R/λ = 0.5	0.8412	0.8674	0.8869	0.9021	0.9144	0.9346	0.9331	0.9403	0.9465	0.9518
R/λ = 1.0	0.8370	0.8756	0.9022	0.9216	0.9362	0.9476	0.9565	0.9637	0.9694	0.9742
R/λ = 2.0	0.8653	0.9133	0.9415	0.9594	0.9712	0.9792	0.9848	0.9888	0.9917	0.9938
R/λ = 5.0	0.9294	0.9712	0.9871	0.9939	0.9970	0.9985	0.9992	0.9996	0.9998	0.9999

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13 ABSTRACT

The calculations presented herein are based on a first-collision model which allows collisions between the free-stream molecules and the molecules which are re-emitted from the surface of the body. The net effect of these collisions is to partially shield the body from the free stream, reducing both the drag and heat-transfer coefficients from the corresponding values experienced in free-molecule flow. Free-stream Mach number is taken to be essentially infinite and the molecules are assumed to be re-emitted from the body surface at the most probable velocity, instead of possessing a velocity distribution. These assumptions enable the distribution of flux to a given body to be expressed as a function of the degree of rarefaction, as represented by the appropriate Knudsen number. This Knudsen number is composed of a characteristic body dimension and the mean free path of the re-emitted molecules relative to free-stream molecules. The integral equation expressing the incident flux distribution on a general body is developed, and solutions are presented for the disk normal to the free stream and for sharp cones of various apex angles, at zero angle of attack. These flux distributions are then integrated to give drag coefficients ratioed to the corresponding free-molecule values.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
molecular flow flux distribution hypervelocity flow mathematical calculations rarefaction cones disks drag coefficients						

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